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ANALYSIS OF MEASUREMENT ERRORS OF CONVECTIVE HEAT-TRANSFER COEFFICIENT USING A THIN-WALLED HEAT-FLUX SENSOR

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A theoretical analysis of the errors is made and it is shown that the convective heat-transfer coefficient in steady conditions on a rotating object of investigation can be measured by thin-walled sensors.

Thin-walled heat-flux sensors [1] have so far been used mainly for measurement of radiative heat fluxes [1, 2]. An account of the method of investigating convective heat transfer in steady conditions by means of these sensors and the results of experimental tests of them are given in [3].

For a further investigation of the potential of sensors we analyzed the errors of measurement of the heat-transfer rate and experimentally tested the sensors on a rotating object — a steel disk of diameter 500 mm and thickness 15 mm. Thirty sensors of the construction illustrated schematically in Fig. 1 were mounted in the disk.

A thin element 1 in the form of a disk of constantan foil 0.05 mm thick was soldered into a copper case 2 in the form of a cylindrical bush with a hole 5 mm in diameter. A copper thermoelectrode 3 of diameter 0.05 mm was welded to the center of the thin element. The copper parts of the sensor — the case 2, the thermoelectrode 3, the wire 4 connected to the case, and the thin constantan element 1 — form a differential thermocouple with junctions at the center of the thin element and at its effective radius r_1.

The interior of the case was filled with heat-insulating material 5 and was closed by a copper plug 6. The investigated surface of the disk was subjected to jets of air from perpendicular nozzles of a radius of 200 mm. On the opposite side heat was delivered to the disk from a stationary electric heater.

The airflow rate, number and diameter of nozzles, angular velocity of the disk, and the distance from the nozzles to the investigated surface were varied in wide ranges. In 20 sensors plastic foam was used as heat insulation, and in the rest Kel-F was used. Heat transfer at the face of the thin element gives rise to a temperature difference between its periphery at effective radius r_1 and the center, which can be determined from the thermal emf of the differential thermocouple of the sensor. The connection between this difference and the heat-transfer coefficient on the face of the thin element is given by the relation

$$I_0(m) = 1/(1 - \Delta\overline{T}_0), \tag{1}$$

obtained in [3] from the equation

$$\frac{\theta_p}{\theta_e} I_0(m\overline{r}) = I_0(m) = \text{const} \tag{2}$$

for the temperature field of a thin element in the case of convective heat transfer. Here the parameter $m = r_1\sqrt{\alpha/\lambda_e\Delta}$, including the required quantity α, characterizes the thermophysical and geometric parameters of the sensor.

The sensors on the object of investigation were placed in groups of six at radii 160, 180, 200, 215, and 232 mm. To increase the accuracy of measurement of α and reduce the effect of the stray emf of the current-removing device the sensors of each group were connected in series and formed differential thermopiles, consisting of six sensors, electrically insulated from the disk by Textolite collars 7. A constantan wire 8 of

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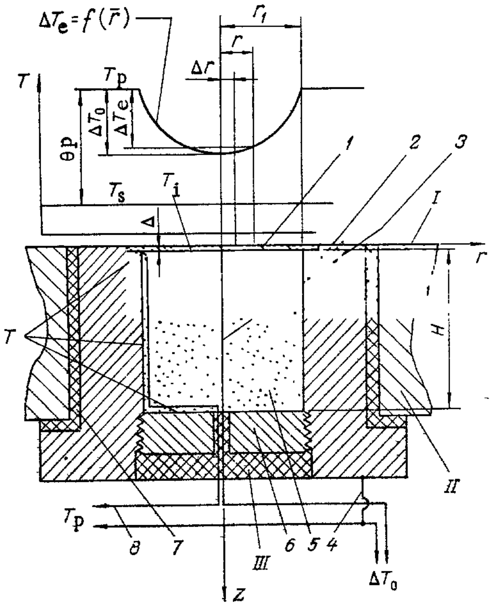

Fig. 1

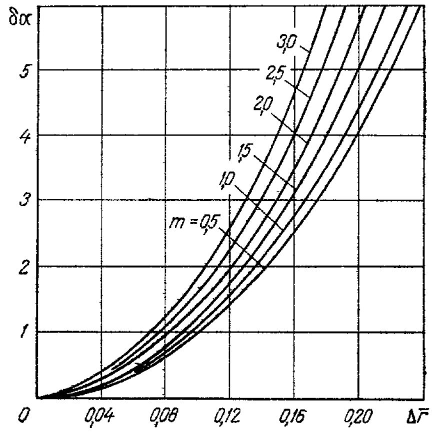

Fig. 2

Fig. 1. Schematic of sensor in object of investigation: I) investigated surface; II) disk; III) epoxy resin.

Fig. 2. Dependence of $\delta\alpha$, %, on $\Delta\bar{r}$.

diameter 0.05 mm, the copper case 2, and the wire 4 formed the thermoelectrode circuit for measurement of the temperature T_p at the periphery of the thin element. The method chosen for investigation of convective heat transfer on the rotating disk was the constant heat-flux method. The main components, measuring system, measurement procedure, and treatment of the experimental data for investigation by the constant heat-flux method were the same as in [4]. We found that the experimental data obtained by investigation of the convective heat transfer of a disk rotating in a closed case and subjected to air jets by measurement with thin-walled sensors by the method described in [3] were in good agreement with the results of investigation by the constant heat-flux method and the dimensionless relations [5] for determination of the local heat-transfer coefficients over the disk radius.

The rms error in investigation by the method of [3] did not exceed 8%. The value taken for the absolute error of a single experiment was the difference in the values of α obtained experimentally and by calculation from the similarity equations [5].

The variable quantities in the experiments lay in the following ranges: $Re = 3 \cdot 10^4 - 2 \cdot 10^5$; $i = 2, 4, 8$; $d = 4, 6, 8$ mm; $U/C_0 = 0.1-0.9$. The metrological potential of the sensor was determined from the deviation of the actual geometric characteristics from the calculated values, the accuracy of measurement of ΔT_0 and Θ_p, and the values of the ignored heat fluxes between the thin element and the sensor components connected to it. To analyze the effect of these factors on the accuracy of measurement of α we turn again to the schematic of the sensor in Fig. 1. It can be shown by using Eq. (2) that the relative error of measurement of α, due to displacement of the point of attachment of the thermoelectrode from the geometric center of the thin element by an amount Δr, can be determined from the equation

$$\delta\alpha = \left[1 - \left(1 - \frac{I_0(m\Delta\bar{r}) - 1}{C}\right)^2\right] 100\%, \tag{3}$$

where

$$C = \left(\frac{1}{\Delta\bar{T}_0} + \Delta\bar{T}_0 - 2\right)(I_0(m) - 1)\, m I_1(m).$$

The relation between $\delta\alpha$ and $\Delta\bar{r}$, calculated from this formula, is shown in Fig. 2 for different values of m. Equation (3) can be useful in the designing of sensors, development of methods of construction, and for analysis of the errors in measurement of α.

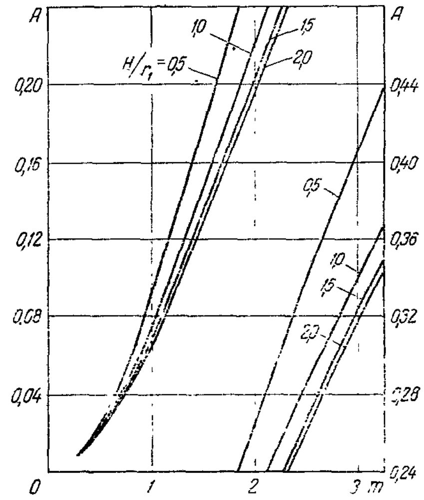

Fig. 3

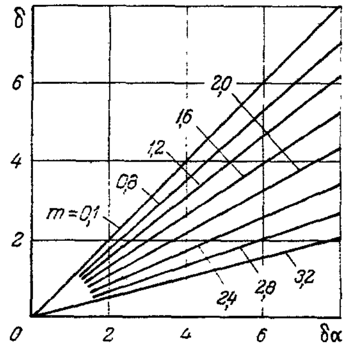

Fig. 4

Fig. 3. Dependence of coefficient A on parameter m and geometric characteristics of insulation.

Fig. 4. Dependence of $\delta\alpha$, %, on δ, %.

Equation (1) for calculation of α was obtained in [3] for the case where heat exchange with the medium occurred only at the face of the thin element and heat fluxes from the insulation and thermoelectrode were ignored. The latter fluxes can be calculated by regarding the insulation and thermoelectrode as a finite cylinder and rod with boundary conditions corresponding to the scheme in Fig. 1 in the steady heat regime. In view of the high thermal conductivity of copper and the small size of the sensor, we can assume that in this case the temperatures of the case 2 and the plug 6 are the same and equal to the temperature T_p on the periphery of the thin element. The temperature distribution over the radius of the thin element is determined from relation (2)

$$\Delta T_e = f(\bar{r}) = \theta_p \left[1 - \frac{I_0(m\bar{r})}{I_0(m)} \right]. \tag{4}$$

Then, assuming $T_p = 0$, we can regard the heat insulation as a finite cylinder with boundary conditions:

$$T_i = T_p = 0 \text{ when } r = r_1,\ H > Z > 0;$$
$$T_i = T_p = 0 \text{ when } Z = H,\ r_1 > r > 0;$$
$$T_i = \Delta T_e = f(\bar{r}) \text{ when } Z = 0,\ r_1 > r > 0$$

provided that $\lambda_i \ll \lambda_e$ and, using the solution of the heat-conduction equation for a finite cylinder [6], we can write the expression for T_i in the form

$$T_i = 2\theta_p \sum_{n=1}^{\infty} \frac{J_0(\bar{r} j_n)\,\mathrm{sh}\,\frac{H-Z}{r_1} j_n}{J_1^2(j_n)\,\mathrm{sh}\,\frac{H}{r_1} j_n} \int_0^1 \bar{r} \left[1 - \frac{I_0(m\bar{r})}{I_0(m)} \right] J_0(\bar{r} j_n)\, d\bar{r}, \tag{5}$$

where j_n are the positive roots of the equation $J_0(r_1 j_n) = 0$.

As a result, we obtain the design equation

$$\Delta Q_i = 2\pi\lambda_i \int_0^{r_1} \frac{dT_i}{dZ_{(z=0)}}\, r dr = 4\pi\lambda_p r_1 \theta_i A, \tag{6}$$

where

$$A = \sum_{n=1}^{\infty} \left(m^2 \,\mathrm{cth}\, \frac{H}{r_1} j_n \right) \Big/ \left(j_n (m^2 + j_n^2) \right). \tag{7}$$

The dependence of coefficient A on the parameter m and geometric characteristics of the insulation, calculated from Eq. (7), is shown in Fig. 3. A similar calculation enabled us to obtain a working formula for the heat flux across the end face of the thermoelectrode (Z = 0) in the form

$$\Delta Q_{\text{T}} = 4\pi\lambda_{\text{T}} \left(\bar{\lambda}_{\text{T}}/\ln \frac{1}{\bar{r}_0}\right)^2 r_0 \Delta T_0 K, \tag{8}$$

where

$$K = \sum_{n=1}^{\infty} \frac{\operatorname{cth} \frac{H}{r_0} j_n}{j_n \left[\left(\bar{\lambda}_{\text{T}}/\ln \frac{1}{\bar{r}_0}\right)^2 + j_n^2\right]}; \tag{9}$$

j_n are the positive roots of the transcendental equation

$$j_n J_1(j_n r_0) + \frac{\bar{\lambda}_{\text{T}}}{r_0 \ln \frac{1}{\bar{r}_0}} J_0(j_n r_0) = 0.$$

Calculations from Eq. (9) show that for sensors with any geometric and thermophysical characteristics $K \leq 1$. Referring to (6) and (8) we can estimate the heat flux from the thermoelectrode by using the ratio

$$\frac{\Delta Q_{\text{T}}}{\Delta Q_{\text{i}}} = \frac{\bar{\lambda}_{\text{T}} \bar{r}_0}{\left(\ln \frac{1}{\bar{r}_0}\right)^2} \frac{K}{A} \Delta \bar{T}_0,$$

from which it follows that the heat flux from the thermoelectrode to the thin element is negligible and the error of measurement of α is determined by the heat influx from the insulation. In view of this, we can write the heat balance equation for an annular section of the thin element with $r = r_1$ in the form

$$|Q| = |Q'| + |\Delta Q_{\text{i}}|, \tag{10}$$

where $Q = \lambda_{\text{e}} F \left.\frac{dT_{\text{e}}}{dr}\right|_{(r=r_1)}$ is the heat flux across the annular section of the thin element when there is no heat influx from the insulation; $Q' = \lambda_{\text{e}} F \left.\frac{dT_{\text{e}}'}{dr}\right|_{(r=r_1)}$ is the same with heat influx from the insulation; T_{e}' is the temperature on the particular radius of the thin element with heat influx from the insulation.

Using relations (2) and (6), we can convert Eq. (10) to the form

$$m \frac{I_1(m)}{I_0(m)} = m' \frac{I_1(m')}{I_0(m')} + \frac{2\bar{\lambda}_{\text{e}}}{\Delta} A, \tag{11}$$

where $m' = r_1\sqrt{\alpha'/\lambda_{\text{e}}\Delta}$ is determined from (1); $m = r_1\sqrt{\alpha/\lambda_{\text{e}}\Delta}$; α' and α are the heat-transfer coefficients measured without and with allowance for heat influx from the insulation.

Equation (11) allows the calculation of α with due allowance for the heat flux from the insulation into the thin element. When there is an air gap between the insulation and the thin element, as was the case in the experiments in [3], the effect of the heat insulation on the accuracy of measurement is negligible and α is calculated from Eq. (1). Operating experience showed, however, that the low mechanical strength of such a sensor limits its applicability. The effect of centrifugal force, vibration, and high pressure of the atmosphere, etc. leads to deformation of the thin element and detachment of the thermoelectrode. The solidly constructed sensor used in the present work and illustrated schematically in Fig. 1 is perfectly reliable. Hence, the values of α for the rotating disk were determined from Eq. (11) with due regard to the heat influx from the insulation. To facilitate the calculations we compiled a table of values of $mI_1(m)/I_0(m) = f(m)$ from $m = 0.36$ to $m = 4$ at steps of 0.01.

Calculations from Eq. (1) and Eq. (11) showed that the error of measurement of α due to heat influx from the insulation over all the experiments was 1-5%, depending on the value of m. The error in calculating α from Eq. (1) is also due to the error of determination of $\Delta\bar{T}_0$, which in the general case is made up of the instrumental error of the measuring circuit, the systematic error of the differential thermocouple of the sensor and the thermocouple of its case (periphery of thin element). In this case the maximum relative error of measurement of α can be calculated from the equation obtained from (1):

$$\delta\alpha = \left[2 \frac{\delta}{D} + \left(\frac{\delta}{D}\right)^2\right] 100\%, \tag{12}$$

where $\delta = \delta\Delta T_0 + \delta\theta_{\text{p}}$; $D = mI_1(m)\left(\frac{1}{\Delta\bar{T}_0} + \Delta\bar{T}_0 - 2\right)$.

Figure 4 shows $\delta\alpha$ as a function of δ, calculated from Eq. (12) for different values of m.

An analysis of the characteristics in Figs. 2 and 4 shows that an increase in m leads to a significant increase in the error of measurement of α. The optimal value of m in the designing of a sensor can be determined by means of Eqs. (3) and (12) or the relations shown in Figs. 2 and 4.

By varying the geometric characteristics of the sensor (r_1, Δ) and selecting materials with appropriate thermophysical properties for the components of the differential thermocouple of the sensor, the value of parameter m can be varied in a wide range. For instance, replacement of the thin constantan element by a copper one, and the copper thermoelectrode by a constantan one, leads to a reduction of m by a factor of four.

The results of the theoretical and experimental investigation carried out indicate that convective heat transfer can be investigated with the aid of thin-walled sensors not only on stationary, but also on rotating, objects of investigation without prior calibration of the sensors against a known heat flux.

The obtained formulas and relations are valid for the case where material with $\lambda_i \ll \lambda_e$ is used as heat insulation.

In application to the design of a sensor they can be used to determine its most rational geometric and thermophysical characteristics from the viewpoint of accuracy of measurement of α in accordance with the experimental conditions, and also to match them with the errors of the measuring circuit.

NOTATION

T, temperature; ΔT_0, ΔT_e, temperature difference between periphery of thin element and center, and between periphery and particular radius; $\theta_p = T_p - T_s$, $\theta_e = T_e - T_s$, temperature difference between periphery of thin element and surroundings, and between thin element on particular radius and surroundings; Δ, thickness of thin element; r_1, r, effective and particular radius of thin element; α, heat-transfer coefficient between thin element and surroundings; λ, thermal conductivity; d, nozzle diameter; Re, Reynolds number calculated from flow-average velocity of outflow of air from nozzles and parameters in front of nozzles; i, number of nozzles; R, radius of position of nozzles; $U = \omega R$, circular velocity of disk at radius of position of nozzles; C_0, velocity of adiabatic outflow of air from nozzles; Δr, displacement of thermoelectrode relative to geometric center of thin element; $\delta\alpha$, $\delta\Delta T_0$, $\delta\theta_p$, relative error of measurement of α of thin element with surroundings, temperature difference between periphery of thin element at effective radius r_1 and center, temperature difference between periphery of thin element and surroundings; H, height of insulation; Z, ordinate along axis of insulation and thermoelectrode; ΔQ_i, ΔQ_T, heat flux into thin element from insulation and thermoelectrode; r_0, radius of thermoelectrode; $F = 2\pi r_1 \Delta$, area of annular section at periphery of thin element. Relative quantities: $\Delta\bar{T}_0 = \Delta T_0/\theta_p$, $\bar{r} = r/r_1$, $\bar{\Delta r} = \Delta r/r_1$, $\bar{\lambda}_T = \lambda_i/\lambda_T$, $\bar{r}_0 = r_0/r_1$, $\bar{\lambda}_e = \lambda_i/\lambda_e$, $\bar{\Delta} = \Delta/r_1$; $J_0(x)$, $J_1(x)$, and $J_2(x)$, Bessel functions of first kind (of zeroth, first, and second orders); $I_0(x)$, $I_1(x)$, modified Bessel functions of first kind (of zeroth and first orders); j_n, roots of transcendental equations. Subscripts: i, insulation; p, periphery of thin element; s, surroundings; T, thermoelectrode; e, thin element.

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